

## UNIT-1

### Extensometers and displacement sensors

#### Experiment:

The special observation mode to confirm or something doubt 801.

#### Stress analysis:

It is an Engineering discipline that determines the stress in materials & structures subjected to static or dynamic forces (or) loads.

**Aim of the analysis:** To determine whether the [element or collection of elements] "STRUCTURE" can safely with stand the specified forces.

Normally the safety load can be measured using F.O.S [factor of safety]

$$= \frac{\text{ultimate stress}}{\text{maximum allow the stress}}$$

This FO.S. given to design engineering for the purpose of design. From the F.O.S the design Analyst calculate design

$$\text{"Design factor " } = \frac{\text{ultimate tensile stress}}{\text{Maximum calculator tensile stress}}$$

Types of load acting on a structure:

- \*Tension
- \*Compression
- \*Shear
- \*Torsion
- \*Bending

Design factor is got by use of these variables

#### 1. What is measurement?

The process of obtaining the magnitude of a quantity such as length or mass relative to a unit of measurement such as meters or kilogram.

- \* The act of measuring or the process of being measured [used]
- \* The system of measuring

#### System & unit is:

- International System of units
- Imperial system
- Metric system

## **TYPES OF MEASUREMENT:**

Generally two measurements

- 1) **VECTOR'S** : have an magnitude [an amount] & a direction
- 2) **SCALAR'S** : have an magnitude but have no direction.

On the basis of S.I units the measure divided & classified into following,

- Linear [length or distance]
- Mass [weight]
- Volume
- Temperature

All measuring instruments have calibrations. These are markings or division in measuring tool.

### **Linear:**

Linear measurements are made using a Metric stick or Metric Ruler.

Measured in meter centimeter millimeter  
                                  ↓                  ↓                  ↓  
                                  m                  cm                  mm

### **Mass:**

Mass measurements are made using a balance

There are several kinds of balance,

- Triple beam balance
- Dial – a gram balance
  
- Electric / digital balance
- Analytical balance

Measured in gram, kilograms, centigrams, milligrams.

### **Volume:**

The volume of any solid, liquid, gas, plasma or vacuum is how much 3-D space it occupies  
Measured in cubic meters, cubic centimeter liters, milliliters.

### **Temperature:**

Temperature measurement using modern scientific thermometers & temperature scales.  
Measured in Fahrenheit, Kelvin, celcius.

## **Principles of measurement:**

The techniques of measurement are of immense importance in most facets of scientific research & human civilization.

Computation with decimals frequently involves the addition or subtraction of numbers do not have the same number of decimal places.

## **Estimation:**

Estimation is the calculated approximation of a result which is usable even if input data may be incomplete or uncertain. It can be computed precisely.

## **Precision:**

The Measurement of a precision depends upon how precisely the instrument is marked. It is important to realize that precision refers to the size of the smallest division on the scale.

Simply we can say, that one instrument is more precise than another does not imply that the less precise instrument is poorly manufactured.

The precision of measurement system also called reproducibility or repeatability



It is degree to which repeated measurement under unchanged conditions show the same result's.

## **Reproducibility:**

It is one of the main principles of the scientific method & refers to the ability of a test or experiment to be accurately reproduced.

## **Repeatability:**

It is the variation in measurement taken by a single, person or instrument on the same item & under the same conditions.

**Accuracy:**

The accuracy of measurement depends upon the relative size of the probable error.

The Accuracy of a measurement system is the degree of closeness of measurements of a quantity to its actual [true] value.

The measurement system is valid if it is both accurate & precise.

$$\text{ACCURACY} = \frac{\text{No of true positives} + \text{no of true negatives}}{\text{no of true positives \& false positives} + \text{false negatives} + \text{true negatives}}$$

$$\text{Precision} = \frac{\text{No of true positives}}{\text{No of true positives} + \text{false positives}}$$

$$\text{Accuracy} = (\text{Sensitivity}) (\text{prevalence}) + \text{specificity} [1 - \text{prevalency}]$$

Accuracy may be determined from sensitivity & specificity provided prevalence.

**Sensitivity:**

$$\text{sensitivity} = \frac{\text{No of true positives}}{\text{No of true positives} + \text{no of false negatives}}$$

**Specificity:**

$$\text{specificity} = \frac{\text{No of true negatives}}{\text{No of true negatives} + \text{no of false positives}}$$

**Example:**

True positives (TP) – sick people correctly diagno as sick

False positives (FP) \_ Healthy as sick

True Negatives (TN) \_ Healthy correctly indentified as healthy

False negatives (FN)\_ Sick people incorrectly identified as healthy

False positives & False negatives also called as Type –I & Type II error

TP → condition present + positive result

FP → condition absent + positive result

FN → condition present + Negative result

TN → condition absent + Negative result

**Example:**

- 1) 3.72 inches or 2417 feet
  - o We can say 3.72 inches is more precise
  - o 2417 feet is more accurate
- 2) 30 seconds or 28 second's
  - 30 second is more accurate & Precise.

**Error:**

It is classified into two types

systematic }  
random } error

Systematic error impacts the accuracy of measurement results

- 1) Faulty instrument
- 2) Faulty measuring
- 3) Personal bias

**Errors to avoided systematic error:**

- 1) Instrument error
- 2) Procedural error
- 3) Personal bias

**Random error's:**

- 1) Least count error
- 2) Mean value of measurement

Percent of Error =  $\frac{\text{probable error}}{\text{Measured value}}$

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# EXTENSOMETERS

## Strain Measurement

### Introduction

Strain gauges are mostly used to measure strains on the free surface of a body. The state of strain at any point on the free surface of a body can be characterized in terms of three Cartesian strain components  $\epsilon_{xx}$ ,  $\epsilon_{yy}$  and  $\gamma_{xy}$  as

$$\epsilon_{xx} = \frac{\partial u}{\partial x} \quad \epsilon_{xy} = \frac{\partial v}{\partial y}$$
$$\gamma_{xy} = \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}$$

Where  $u$  and  $v$  are the displacement components in  $x$  and  $y$  directions respectively. These equations suggest that if the two displacements  $u$  and  $v$  can be measured at all points on the surface of a body, strains at any point on the surface can be determined.

It is seen from Eq. (16.1) that the Cartesian strains are actually the slopes of the displacement surfaces  $u$  and  $v$ . For precision in the estimation of the slopes of the displacement surfaces, the in-plane displacements  $u$  and  $v$  should be determined quite accurately. However, particularly for small elastic strains, the in-plane displacements are exceedingly small. No versatile and easy method is yet available for the direct measurement of these displacements over the entire surface of a body. This difficulty is overcome partially by using a strain gauge to measure the change in the distance between two points on the surface of the body due to straining. This change in length is converted to axial strain by the following relationship:

$$\epsilon_{xx} = \frac{\Delta u}{\Delta x}$$

Here  $\Delta u$  is the change in length over a distance or the gauge length,  $\Delta x$ . It is to be noted that the strain measured in this manner represents only the average strain over the gauge length,  $\Delta x$ . The magnitude of error in the strain measured this way depends on the strain gradient along the gauge length  $\Delta x$  and the length  $\Delta x$ . This aspect is discussed further in Sec.

Strain gauges of all types are essentially devices that sense the change in length, magnify it and indicate it in some form. They can be classified into broadly five groups on the basis of the physical employed for the magnification of change in length.

- (i) Mechanical
- (ii) Optical
- (iii) Interferometric type
- (iv) Electrical
- (v) Pneumatic, and
- (vi) Acoustical
- (vii) Magnetic
- (viii) Scratch gauge
- (ix) Photoelastic gauge

## Mechanical Strain Gauges:

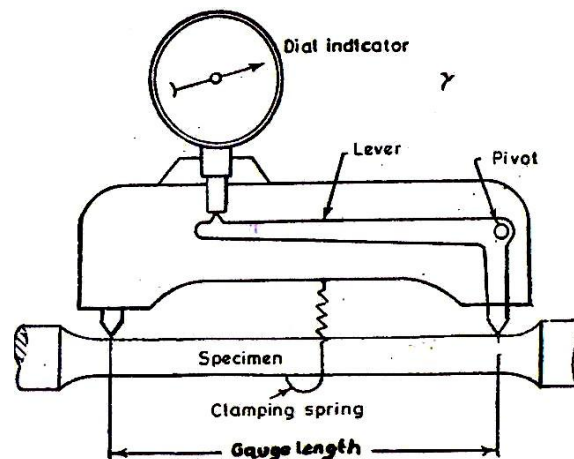
- i) Wedge and screw magnification
- ii) Simple mechanical lever magnification
- iii) Compound magnification system
- iv) Compound lever magnification system
- v) Magnification by rack and pinion
- vi) Combined lever, rack and pinion magnification

### Compound magnification system

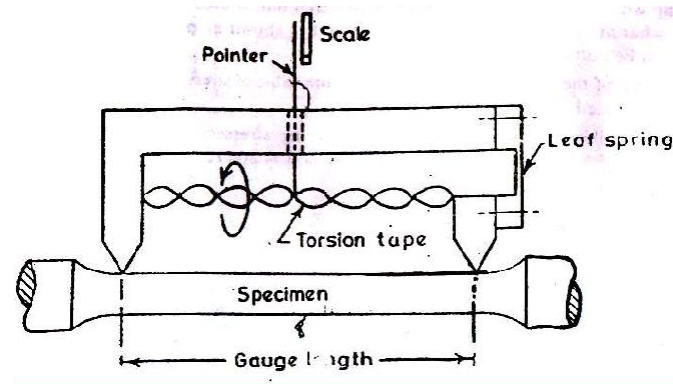
#### (a) Huggenberger and CEJ Extensometers

These mechanical devices are generally known as extensometers and are used to measure strain under static or gradually varying loading conditions. An extensometer is usually provided with two knife edges which are clamped firmly in contact with the test component at a specific distance or gauge length apart. When the test component is strained, the two knife edges undergo a small relative displacement. This is amplified through a mechanical linkage and the magnified displacement or strain is displayed on a calibrated scale.

The Berry strain gauge (Fig. 16.1) uses a system of a lever and dial gauge to magnify the small displacement between the knife edges. It can measure strains down to 10 microstrain over a 50 mm gauge length. The mechanical amplifying element in the CEJ extensometer is a twisted metal strip or torsion tape stretched between the knife edges.



**Figure: Berry Strain Gauge**

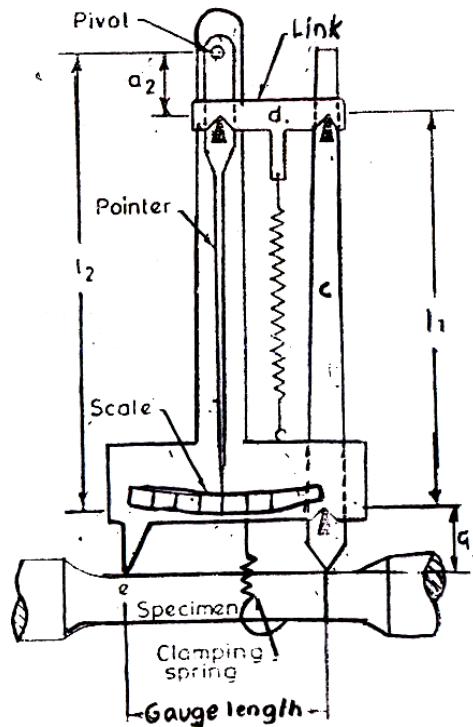


**Figure: Johansson extensometer**

Half the length of this strip is twisted in one direction while the other half is twisted in the opposite direction. A pointer is attached at the centre. The displacement of the knife edges, i.e. stretching of the torsion tape is converted into a highly amplified rotational movement of the pointer. The CEJ extensometer can measure strain with a sensitivity of 5 micro strain over a gauge length of 50 mm.

**Compound lever magnification system**

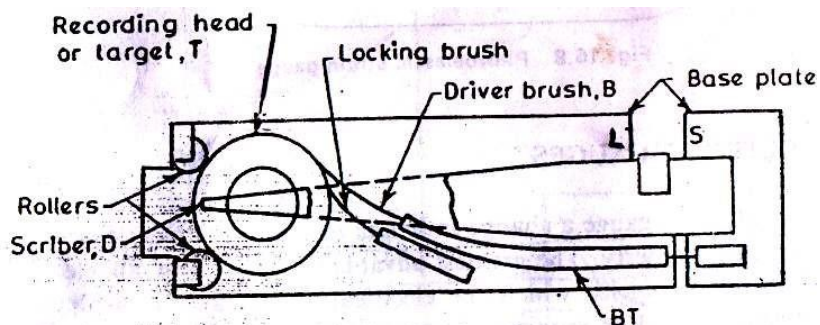
In the Huggenberger extensometer (Fig. 16.3) a set of compound levers is used to magnify the displacement of the knife edges. The extensometer is highly accurate, reliable, light-weight and self-contained. The movable knife edge (*f*) rotates the lever *c* about the lower pivot. The lever *c* in turn rotates the pointer through the link *d*. The magnification ratio is given by  $l_1 l_2 a_1 a_2$ . Extensometers with this ratio varying between 300 and 2000 and with gauge lengths in the range 6.5 to 100 mm are available. The sensitivity of these extensometers could be as high as 10 micro strain. It is well suited for applications where its unusually large height does not pose problems of instability in mounting.



**Figure: Huggenberger extensometer**

**(b) Scratch Gauge**

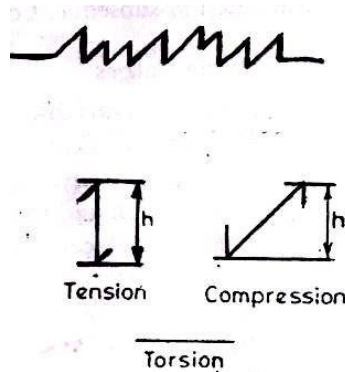
The scratch gauge is a self-contained compact device providing a permanent record of displacement over a period of time. In this gauge



**Figure: Scratch gauge (Prewitt Associates, USA)**

the relative displacement between two stainless steel base planets  $L$  and  $S$  secured to the test component causes a scribe  $D$  to scratch sharply the actual component deformation on a small

brass (target,  $T$ ). The target is held in position by two tiny rollers and two stainless steel brushes. The free end of the long driver brush  $B$  engages a peripheral groove of the target. It is also guided in a bent tube  $BT$ . When a tensile deformation is removed or a compressive deformation is produced, the plates  $L$  and  $S$  move towards each other. This causes the driver brush  $B$  to rotate the circular target by a small amount. However during a tensile deformation the driver brush  $B$  just slides back in the target groove without rotating it. Thus tensile movements scribe a line parallel to the gauge axis (Fig. 16.5). Compressive movements and removal of tensile strain scribe a line at approximately  $45^\circ$  to the gauge axis. The height  $h$  of the recorded data is the product of the strain and gauge length. The traces on the target are evaluated by viewing them with a microscope having a calibrated eye-piece scale. The minimum strain that a scratch gauge can sense is about 100 micro strain. The gauge lengths of these gauges are rather large.



**Figure: Scratch gauge record**

The scratch gauge is compact in size and weighs less than 30 g. It can be attached to almost any surface with clamps or screws or adhesive bonding. It can measure stresses under all types of loading-static, fatigue or shock. It can be used to record stresses in all types of environments- room and elevated temperatures, under water, under radiation, etc.

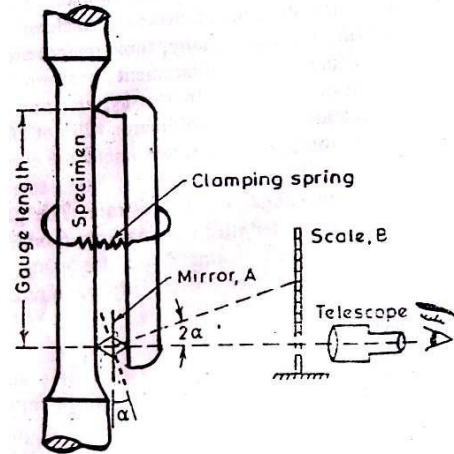
## Optical Gauges

### (a) Mechanical-Optical Gauges

In mechanical-optical gauges a combination of mechanical and optical levers are used to amplify the relative displacement between the knife edges. The moving knife is pivoted.

So that it rotates while undergoing displacement.

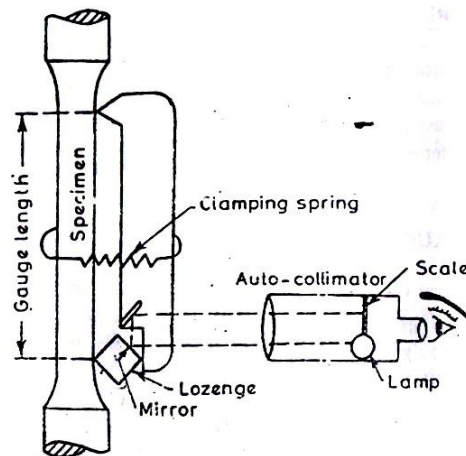
The principle of the signal mirror system is illustrated in Fig. 16.6. The pivoting knife edge carries a mirror  $A$ . The reflection of an illuminated scale  $B$  in this mirror is viewed through the observing telescope. Any deformation of the structure to which this gauge is fixed, rotates the mirror  $A$  and thereby brings different portion of the scale into view.



**Figure: Martens optical gauge**

Thus the change in the reading on the scale is directly proportional to the deformation being measured.

A schematic diagram of the Tuckerman optical gauge and the autocollimator used with it is given in Fig. 16.7. The autocollimator carries both the source of a parallel beam of light and an optical system with reticle to measure the deflection of the reflected ray. A tungsten-carbide.



**Figure: Tuckerman optical gauge (American Instrument Co., Inc)**

Rocker (lozenge) functions as the moving knife edge. One face of this lozenge is polished to function as a mirror. The rotation of the lozenge resulting from a deformation of the structure deflects the incident parallel light beam back to the measuring reticle. Actually, three images are visible on the reticle- one giving the measured displacement or strain and the other two helping the alignment of the gauge. In this system, any relative motion between the component and the autocollimator will not affect the measurement. Also, errors due to rotation of the extensometer are eliminated in this system. The sensitivity of the Tuckerman gauge is 2 micro strain. The gauge is available with a wide range of gauge lengths, starting from 6 mm. It can reliably measure both static and dynamic

strains. With the gauge, cyclic strains up to a frequency of 180 c/s have been successfully measured.

### Photoelastic Strain Gauges

A Photoelastic Strain Gauge (Fig. 10.8) essentially consists of: (i) a strip of plastic with a reflective backing containing a “frozen-in” fringe pattern of equally spaced fringes, (ii) a sandwich sheet of a Polaroid and a quarter-wave plate covering the plastic strip, and (iii) a graduated scale for measurement. This gauge when bonded to a test component will indicate visually and quantitatively the presence of strain through the movement of the residual fringe pattern. Usually a principle strain difference of 1000 microstrain causes one fringe to move a distance equal to the fringe spacing. If one can read the fringe position to one-twentieth of the fringe spacing, a sensitivity of 50 microstrain can be obtained.

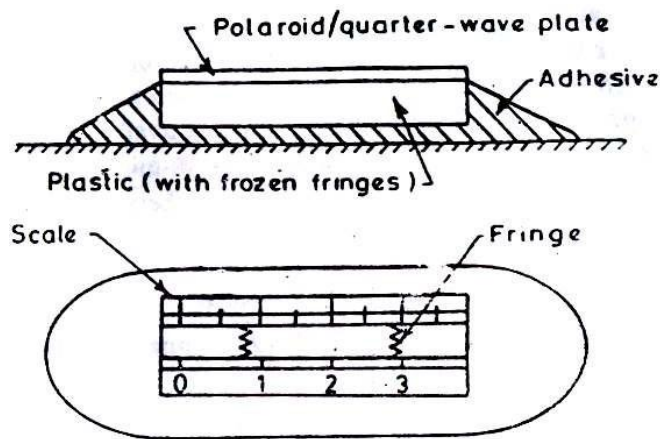


Figure: Photoelastic strain gauge

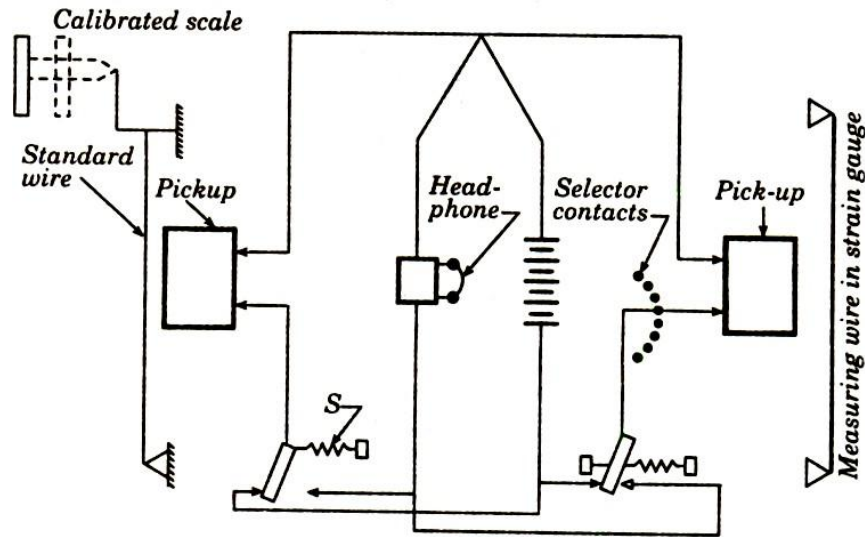
### Acoustical Strain Gauge:

The vibrating wire or acoustical gauge consists essentially of a steel wire tensioned between two supports a predetermined distance apart. Variation of the distance alters the natural frequency of vibration of the wire and this change in frequency may be correlated with the change in strains causing it. An electro-magnet adjacent to the wire may be used to set the wire in vibration and this wire movement will then generate an oscillating electrical signal. The signal may be compared with the pitch of an adjustable standard wire, the degree of adjustment necessary to match the two signal frequencies being provided by a tensioning screw on the standard wire. Calibration of this screw allows a direct determination of the change of length of a measuring gauge to be made once the standard gauge has been tuned to match the frequency of the measuring wire.

The visual display produced on a CRO renders adjustment easier. Tuning is now more usually accomplished by feeding the two signals into the two pairs of plates of an oscillograph and making use of the Lissajous figure formation to balance the frequencies. Matching of the tones is simplified and made more accurate by tuning out the beats which results when the vibration frequencies of two wires are nearly the same, which can be compared by using earphones.

The fundamental frequency of a stretched wire may be estimated from the expression.

$$f = \frac{1}{2L} \sqrt{\frac{P}{m}} = \frac{1}{2L} \sqrt{\frac{(E\delta L) / L A}{m}}$$



**Figure: Acoustical Strain Gauge**

Where

A = cross – sectional area of vibrating wire

E = Young's modulus of wire material

L = length of vibrating wire

m = mass per unit length of the wire

P = tensioning force in the wire

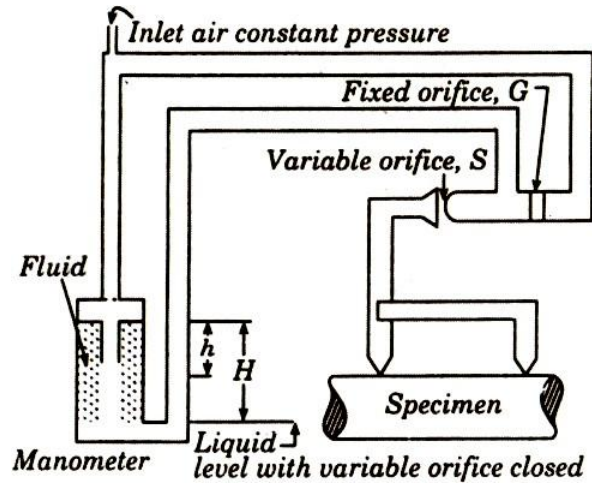
$\delta L$  = increment in length of the vibrating wire.

Figure: Shows an acoustical gauge developed by Dr. O. Schaefer about 1933. The sensitivity of this gauge is very high, with possible determinations of displacement of the order of  $0.25 \mu \text{ cm}$ . The range is limited to about  $1/1000$  of the wire length. The gauge is temperature sensitive unless the thermal coefficients of expansion of the base and wire are closely matched over the temperature range encountered during a test.

### **Pneumatic Strain Gauge:**

The principle of operation of an air or pneumatic gauge depends upon the relative discharge of air between a fixed orifice and a variable orifice. Fig. shows a pneumatic gauge.

Air under constant pressure H, flows through two orifices placed in series. The pressure h which prevails between these two orifices is a function of the ratio of their areas. The fixed orifice G is called the nozzle and the second orifice S, which is smaller, is called the exhaust orifice and is of variable area of cross-section.



**Figure: Pneumatic strain gauge**

As a result of it, the pressure  $h$  serves to measure the dimension of  $S$ . Air after passing through the orifice  $G$ , strikes the top plate and is vented to the atmosphere. The flow of air through the two orifices in series must be equal if incompressibility is assumed. This assumption is practically valid as the pressures are quite low. Let,

- $A_G$  = cross – sectional are of nozzle orifice  $G$
- $A_S$  = cross – sectional area of discharge orifice  $S$
- $C_G, C_S$  = coefficients of contraction for the orifices
- $\rho$  = density of air
- $g$  = acceleration due to gravity

Since the flow through each orifice is the same, hence

$$C_G, A_G \sqrt{\frac{2g(H-h)}{\rho}} = C_S A_S \sqrt{\frac{2gh}{\rho}}$$

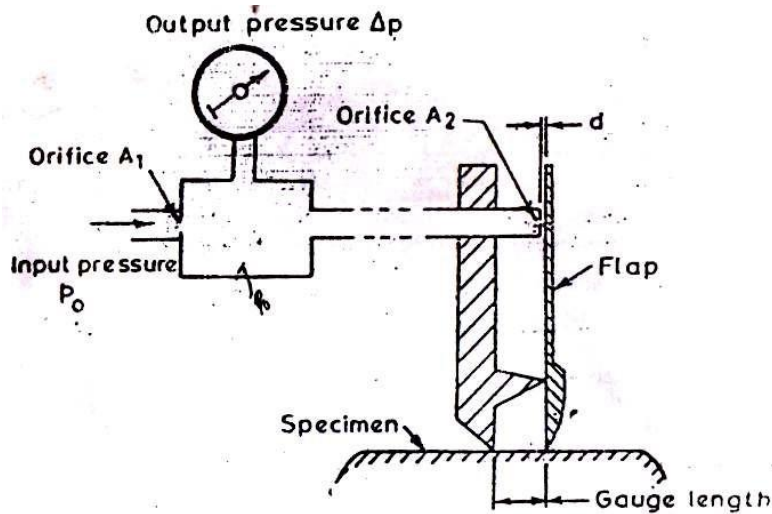
When  $C_S = C_G = C$ , then

$$h = \frac{H}{1 + (A_S / A_G)^2}$$

Figure shows the basic arrangement in a pneumatic strain gauge. Air at constant pressure flows through two orifices of cross – sectional areas  $A_1$  and  $A_2$ . The area  $A_2$  of the variable area orifice is a function of the gap  $d$  which varies as the distance between the knife edge changes. The pressure  $\Delta p$  built up in the chamber is approximately given by

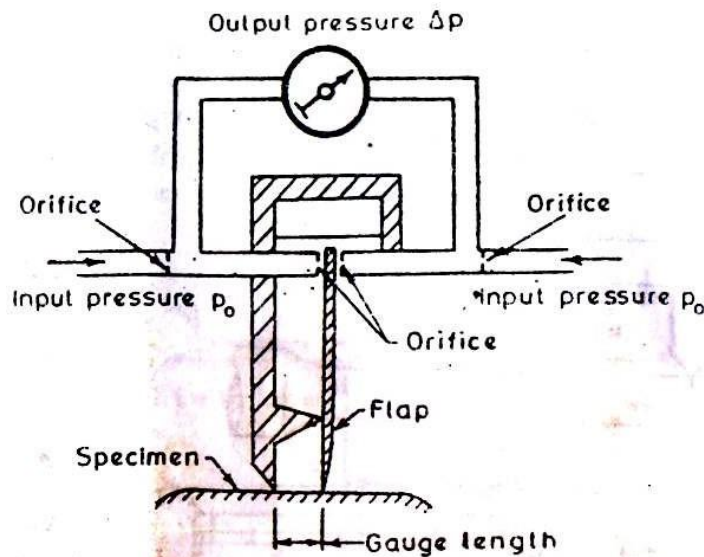
$$\Delta p = \frac{p_0}{1 + \left(\frac{A_2}{A_1}\right)^2}$$

Thus the relationship between  $\Delta p$  and the displacement of the extensometer  $d$  is nonlinear. However, with proper design this non linear characteristics of the gauge can be minimized and a nearly linear characteristic can be obtained over a narrow range of displacement.



**Figure: Pneumatic strain gauge – Single pressure output**

Better linearity can be obtained in the arrangement shown in figure. Magnifications up to 100,000 and gauge lengths as small as 1 mm are possible to achieve in these gauges.



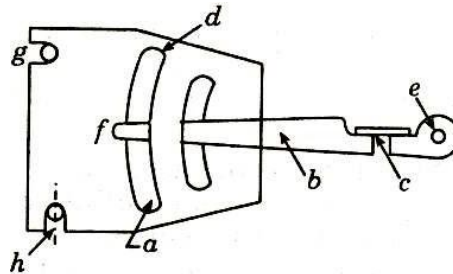
**Figure: Pneumatic strain gauge – Differential pressure output**

Pneumatic gauges are sensitive, robust and reliable. They are suitable for both static and dynamic strain measurements

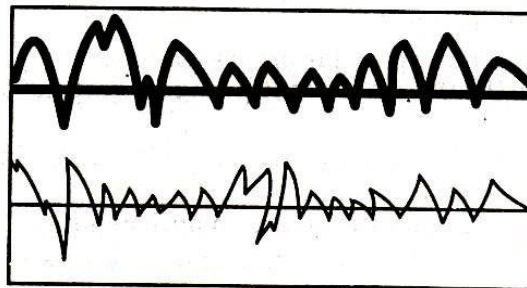
When the specimen is loaded, the distance between the two gauge points changes. This elongation is transmitted through the level's system to the pneumatic gauge, where it changes the gap between orifice S and the top plate, this changing the area  $A_s$  in direct proportion to the strain. From Eq. it is obvious that the manometer reading varies as a quadratic function of the strain. However, it has an inflection point when  $h/H = 3/4$  or  $A_s/A_G = 0.58$ . Hence, for values in this neighborhood, the relation is very nearly linear. Multiplication factors of 100,000 are possible with this type of pneumatic amplification.

### Scratch Gauge:

Of all the gauges available, perhaps the most ingenious because of its simplicity, is the scratch gauge. The instrument consists of two parts; a target, and a scratch arm. These parts may be secured to the test piece by screw, solder, or clamp applied at  $g$  and  $e$ . The target is a small plate with a chromium-plated surface and includes a raised clip arm 'a'. The scratch arm  $b$  is pivoted at the elastic hinge  $C$  and carries, at  $f$ , several grit particles embedded in cured rubber. Motion between  $e$  and  $g$  is recorded as scratches made by the grit particles  $f$  on the chromium plated target. Propulsion of the scratch arm across the target is accomplished by the spring action of the elastic hinge  $C$ .



(a) Scratch recording system



(b) Magnified strain record

Figure: Scratch gauge

The cross motion is regulated by the pressure of clip  $a$  on arm  $b$ . This pressure develops sufficient static friction to restrain the arm, but cross motion is permitted by the smaller sliding friction, which results from relative motion of the target and arm and the test piece is strained. The rate of the scratch arm cross travel is a function of the sliding motion occurring, rather than of time. It can be controlled by variation of the clip pressure and the thickness of the spring hinge. The several grit particles scratch patterns of varying depth, depending on the alignment of

target and arm. Of course, the most clearly defined pattern is used for interpreting the record. The base line is established by moving the arm across the target while the test piece is in an unloaded condition. The measurements of deformation indicated by the scratches on the target, is accomplished by means of a microscope. Figure (b) shows the strain record.

This gauge is used for measuring deformation of rail – road rails because of the load caused by a passing train. Strain measurements can be made inside pressure vessels, inside moving mechanisms, under water etc. Dynamic strains pattern on analysis can be separated into various harmonics of a vibratory deformation. However, this gauge cannot be applied to small structures or finished surfaces, and extreme care has to be exercised in measuring the scratch record to obtain any degree of precision. The gauge is also little affected by its own inertia forces. Readings may be estimated to 0.0025 mm.

## **Electrical Strain Gauges:**

### **Introduction:**

An electrical strain gauge is a device in which a change in length produces a change in some electrical characteristics of the gauge.

The electrical strain gauges may be classified as follows:

- (a) The inductance or magnetic strain gauges.
- (b) The capacitance strain gauges.
- (c) The electrical resistance strain gauges.

Out of these three types of gauges, the resistance strain gauges (RSG) have become more popular and reliable. Hence, in this chapter, we shall study the first two types of gauges briefly and lay more emphasis on the resistance gauges.

The inductance type of strain gauges in which the strain is measured as a change in the magnetic field, was developed as a strain gauge about 1930 by Shamberger. Since then this gauge has been used for various applications, particularly in motion measurements. An important application of inductance type of gauge is the linear variable differential transformer, developed by Schaevitz about 1947.

The capacitance type of gauges have found very little use as strain gauges and are not commercially available. However, these type of gauges have found applications as transducers to measure pressure, force and displacement.

In 1856 Lord Kelvin reported that the electrical resistance of certain wires varied with the tension to which the wires were subjected. Bridgeman in 1923 confirmed Kelvin's results in a series of tests involving wires under hydrostatic pressure. Little use was made of this knowledge until after 1930, when attempts were made to apply the phenomenon of strain sensitivity in wires to the actual measurement of strain in other bodies. The first use of this principle for strain measurements was made by Carlson and Eaton about 1931. A non metallic, unbounded resistance gauge was however developed and used by Mc – Collum and Peters in 1924. A non – metallic, bonded resistance gauge was developed by Bloach in 1935.

The bonded wire metallic strain gauge was developed independently and almost simultaneously in 1938 by Simmons at the California Institute of Technology and Ruge at the Massachusetts Institute of Technology, now commercially known as the SR 4 gauges, and marketed by Baldwin – Lima – Hamilton Corporation of U.S.A. Since then the activity in this field has been on the increase and a lot of improvements has been made in these gauges. During the 1950's considerable attention was given to the foil – type strain gauges. Much of the credit for development and acceptance of this type of gauge goes to Bean, Saunders and Roe. Currently the foil gauges have largely displaced the wire gauges.

A uniquely constructed weld able wire filament strain gauge has been developed recently for application in many hostile environments and installation by Ailtech (U.S.A)

### Electric Inductance Strain Gauges:

An electric inductance gauge is a device in which the mechanical quantity to be measurement produces a change in the magnetic field, and hence in the impedance, of a current – carrying coil. The impedance of a coil depends on its inductance and on its effective resistance, and either or both of these quantities can be made sensitive to the mechanical quantity being measured. The inductance which is changed can be either the self inductance of the coil or its mutual inductance with respect to another coil. Depending upon the method of varying the impedance, electric – inductance gauges may be classified as follows:

1. Variable – air – gap gauges. In which the reluctance of the magnetic field is varied by changing the air gap.
2. Movable – core solenoid gauges. In which the reluctance of the magnetic circuit is varied by changing the position of the iron core in the coil.
3. Eddy current gauges. In which the losses in the magnetic circuit are varied by changing the thickness or position of the high – loss element inserted in the magnetic field.
4. Magnetostriction gauges. In which the reluctance of the magnetic circuit is varied by changing the stress in the magnetic core of the coil.

The impedance of a coil to the passage of alternating current is given by the expression:

$$Z = \sqrt{(2\pi fL)^2 + R^2}$$

Where

Z = impedance in ohms

f = frequency in hertz

L = inductance of the coil in henrys

R = resistance component in ohms.

In general, R is negligible as compared to L and the impedance varies almost in proportion to the inductance. The inductance of a variable – air – gap gauge is given by

$$L = \frac{8.1026N^2}{\frac{l_i}{\mu a_i} + \frac{l_a}{a_a}} \times 10^{-8}$$

Where

N = number of turns.

$l_i$  = length of iron magnetic circuit, cm

$l_a$  = length of air gap, cm

$\mu$  = permeability of magnetic material at the maximum alternating flux density

$a_i$  = cross – section of iron,  $\text{cm}^2$

$a_a$  = cross – section of air gap,  $\text{cm}^2$

If the value of  $\mu$  is sufficiently large, then  $\frac{l_i}{\mu a_i}$  is negligible as compared to  $\frac{l_a}{a_a}$  and we find that

$$L = \frac{a_a}{l_a} \times 8.1026 N^2 \times 10^{-8}$$

The relationship between the voltage applied across a coil and the flux density in its core is

$$E = 0.10667 B a N f \times 10^{-8}$$

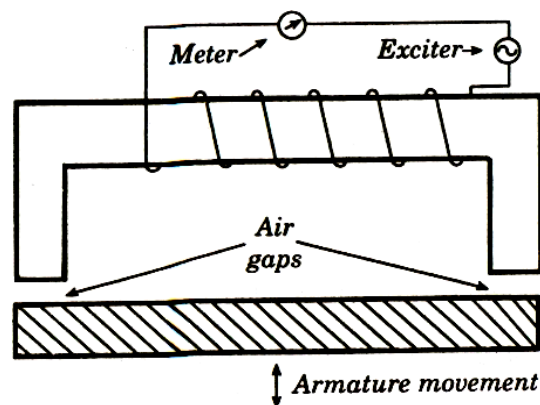
Where

E = voltage

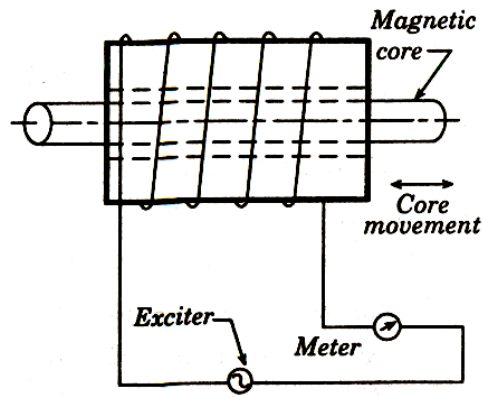
B = flux density in lines per square cm

a = cross section of core,  $\text{cm}^2$ .

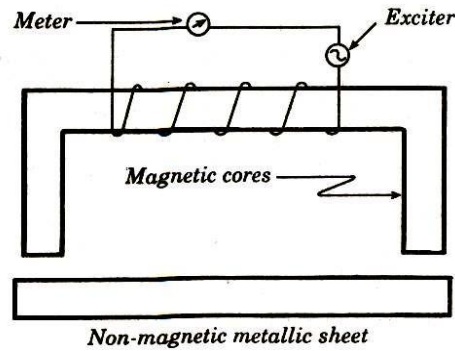
Thus, we find that under ideal conditions, the impedance of an iron – core coil varies inversely with the length of the air gap in the magnetic circuit. If the motion to be measured is a large percentage of the initial air gap, very large change in impedance can be produced and large amounts of electric energy become available. Therefore, the variable – air – gap gauge is one of the best – known methods of converting small motions into high energy electric signals. For large motions, it is more advisable to use the moving – core solenoid. Eddy – current gauges find applications in special fields, such as the measurement of motion and of the thickness of non ferrous sheets. Magnetostriction effect is small in most commercial magnetic irons but is large in nickel and some nickel – iron and cobalt – iron alloys. Figure shows circuits for some of the gauges. These gauges are placed in one arm of an inductance bridge with either a voltmeter or CRO to indicate the out – of – balance potential to the bridge. The bridge is supplied with an alternating current of about 1000 hertz for static strain measurements. For dynamic strains, the frequency of current source must be 20 to 30 times the frequency of the strain being measured. One serious difficulty is that magnetic forces set up across the air gap frequently give rise to serious vibrations in the structure. These gauges cannot be applied to light structures. Large strains cannot be measured as the strain – inductance relationship is linear only over a small range of strain. These gauges are weighty, bulky and susceptible to magneto – mechanical resonance.



(a) Variable air gap gauge



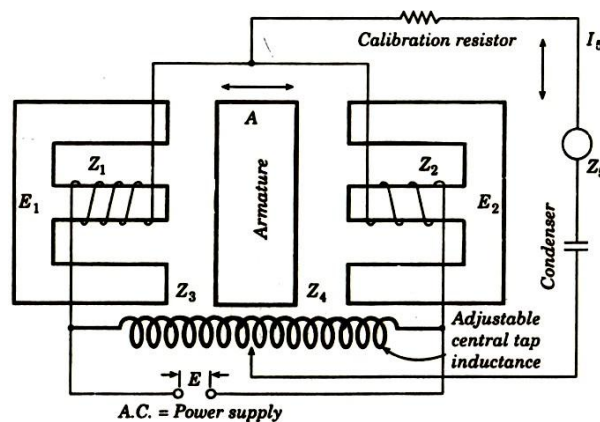
(b) Moving coil solenoid gauge



(c) Eddy current gauge

**Figure: Inductance strain gauges**

Figure shows the basic impedance bridge circuit.  $E_1$  and  $E_2$  are the laminated iron cores and  $A$  is the armature. When the armature  $A$  moves the air gap between  $A$  and  $E_1$  increases and that between  $A$  and  $E_2$  decreases or vice versa. This changes the reluctance of the magnetic paths in  $E_1$  and  $E_2$  and consequently changes the impedance of the two coils which are wound on them. Let  $Z_1, Z_2, Z_3$  and  $Z_4$  be the impedance of the gauge coils and balancing elements.  $Z_5$  is the impedance of the instrument circuit. In order to reduce the voltage across  $Z_5$  to zero, both the resistive and the reactive components of the bridge legs must be balanced. Both of the following conditions must be fulfilled.



**Figure: Basic impedance bridge circuit**

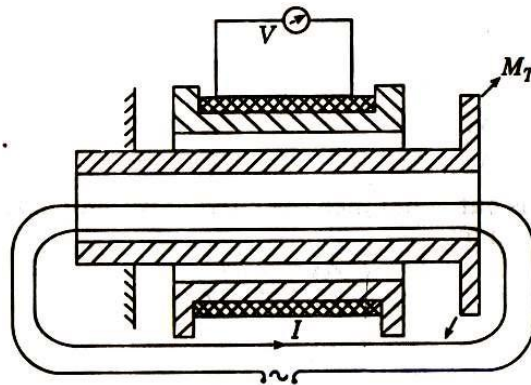
$$\frac{R_1}{R_2} = \frac{R_3}{R_4}$$

And

$$\frac{X_1}{X_2} = \frac{X_3}{X_4}$$

**Electromagnetic Strain Gauge:**

If a magnetic bar is loaded by a torsional moment, a voltage is recorded by a galvanometer connected to a coil through which the bar is put. An electromotive force is induced in the coil of the electromagnet which depends on the torsional moment acting on the core of the electromagnet which is twisted. This is known as Wiedemann’s effect. The factors which influence the magnitude and linearity of the induced electromotive force are: the degree of saturation of magnetic field, the geometry of the attachment to the structure, the frequency of power supply, the size of the tube, the number of turns of the wire wound round the tube, the material of the pipes and its condition. The electromagnetic strain – gauge is shown in figure. The pick up unit, measuring the longitudinal component of the magnetic flow, consists of a pick up coil and a low drain a.c. voltmeter. The influence of the pick up unit depends on the non – inductive resistance  $R_m$  of the measurement device, on the inductive resistance  $\omega l$  of the coil, on the non – inductive resistance  $R_c$  of the coil and on the slenderness ratio  $r = l/D$  of the tube. The mathematical formula expressing the influence of the measurement unit on the variation of the induced electromotive force or the actual, i.e., measured slope of the characteristics  $K_m$  is



**Figure: Electromagnetic strain gauge (Wiedemann’s effect)**

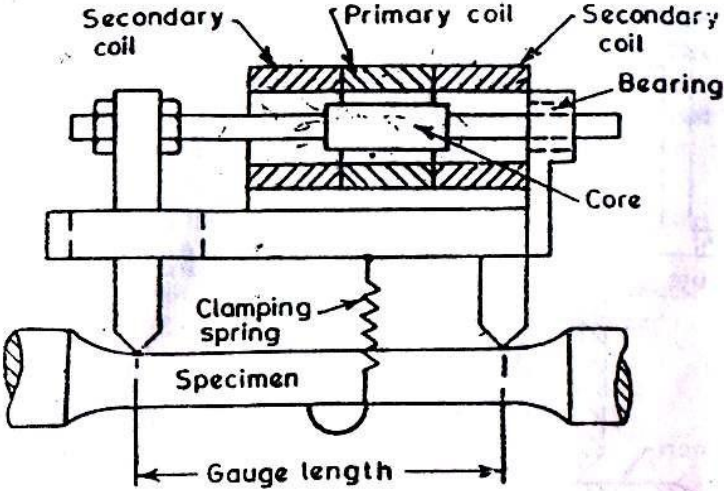
$$K_m = K_0 \frac{R_m}{R_c} \frac{x}{1 + \left[ \frac{\omega l}{R_c} \right]^2}$$

Where  $K_0$  = ideal slope of the characteristic, for  $r = 0.8$ .  
 $x$  = ratio of induced electromotive force for general  $r$  to the voltage  $E_m$  for  $r = 0.8$ .

**Linear Variable Differential transformer**

Of the various types of variable inductance gauges, the linear variable differential transformer (LVDT) is well known for the measurement of displacement. A variety of

transducers for measurements of strain, displacement, pressure, acceleration and force have been built with LVDT as the sensing element. In a transducer with LVDT as the strain-sensing element, the base carrying the primary and secondary coils is attached to one knife edge while the movable magnetic core is connected to the order (Fig.16.9). The centre primary coil is fed from an AC supply. The two balanced secondary windings on either side of the primary coil, connected together in phase opposition function as pick-up coils. The output from the LVDT is theoretically zero when the sliding magnetic core is placed midway



(a) Gauge

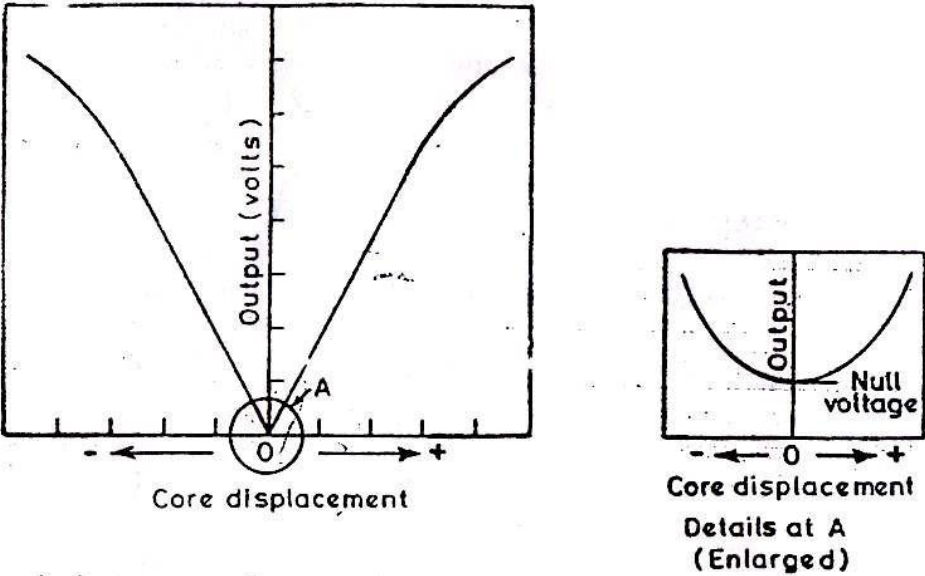


Figure: Linear variable differential transformer gauge

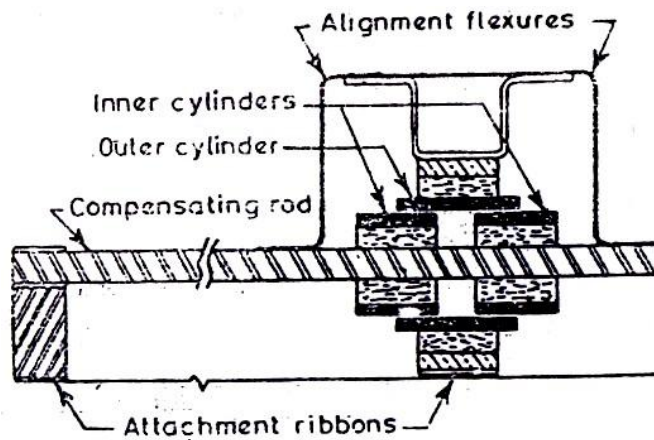
Between the secondary coils. Movement of the core in one direction away from the null position produces an output alternating voltage proportional to the displacement from the centre. Displacement of the core in the opposite direction will produce an output 180° out of phase with the first output. The phase angle has to be determined for finding out the direction of displacement of the core.

The frequency of the applied ac voltage, i.e. the carrier frequency, limits the dynamic response of the LVDT. The frequency of the signal being measured should be less than about one – tenth of the carrier frequency. Further, the dynamic response of the LVDT is restricted by the mass of the core and the supporting mechanical components.

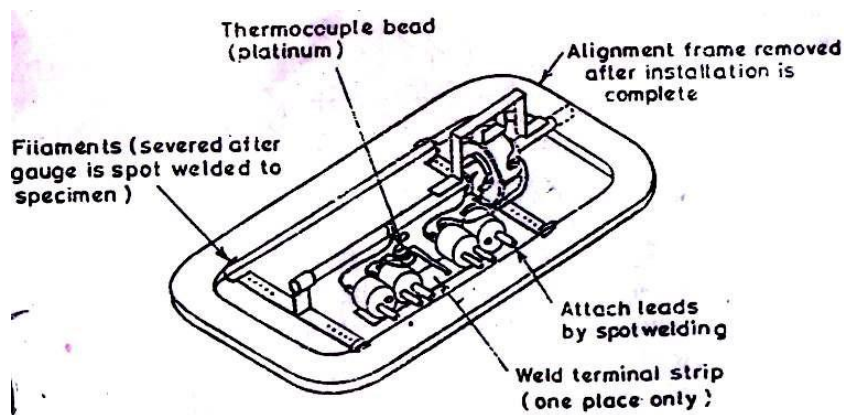
The LVDT requires a driving force of the order of a fraction of a gram to move the core in the coils. It can be used over a wide range of temperature – below zero to elevated temperatures. It provides a high – level output. The sensitivity of LVDT usually lies in the range 0.02 to 0.15 V/mm displacement per volt of excitation applied to the primary coil. A point to be noted is that the performance of the LVDT can be severely affected by the presence of metal masses and stray magnetic fields in its vicinity. The size and mass of the LVDT and the problem of mounting through knife edges rather restricts its use in strain gauge work.

**(b) Capacitance Strain Gauges:**

The capacitance of a condenser can be varied by either varying the distance between the condenser plates or by varying the area. In a capacitance strain gauge the displacement. Resulting from the strain in the test



(a) Cross – section view



(b) Gauge in a alignment frame

**Figure: Hitec capacitance gauge (Hitec Corporation, USA)**

Component varies its capacitance either by varying the distance between the condenser plates or by varying the area between the plates. In the capacitance gauge shown schematically in figure, capacitance changes occur due to axial sliding of an outer cylinder relative to two concentric inner cylinders. Temperature compensation is achieved by using a compensating rod fabricated from a material with the same thermal characteristics as the test component. It functions satisfactorily at temperatures up to about 800°C. With a refined measurement technique, it can measure  $\pm 20,000 \mu\epsilon$  with a sensitivity of  $1 \mu\epsilon$ .

The electrical capacity between parallel plates is given by:

$$C = \frac{8.86 \times 10^{-3} KA(N-1)}{h}$$

Where

C = capacitance in pico farads.

K = dielectric constant of the medium between the two plates.

N = number of plates

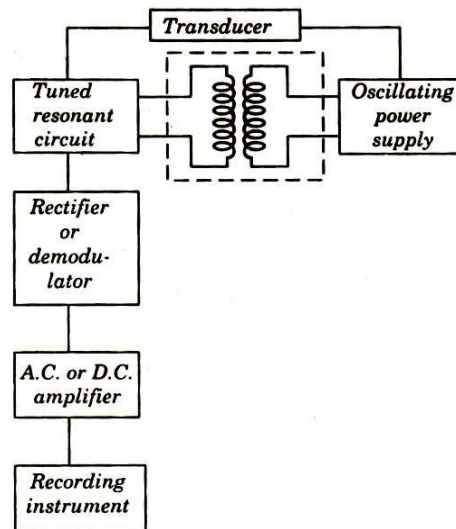
h = distance between plates, mm.

Now  $\frac{C}{dh} = \frac{8.86 \times 10^{-3} KA(N-1)}{h^2} = -\frac{C}{h}$

Strain  $\epsilon = \frac{\Delta h}{l_0}$

Hence  $\epsilon = \frac{h}{l_0} \frac{\Delta C}{C}$

Where  $l_0$  is the gauge length



**Figure: Simplified diagram of a capacitance transducer circuit**

Thus the capacitance of a condenser may be changed either by changing the spacing between the condenser plates or the condenser plate area may be changed. The variation in the capacitance because of a change in the plate spacing while the change in capacitance resulting from a change in area is linear for large changes in the plate area. Two types of circuits may be used to measure the change in capacitance of a gauge. In the first methods, a capacitance bridge is supplied with an a.c., the out – of – balance of the bridge, because of a change in the capacitance of the gauge, is measured by either a voltmeter or CRO. In the second method, the capacitance gauge may be placed in a circuit, oscillating at resonance. As the capacitance of the condenser changes as a result of strain the frequency of oscillation of the circuit changes. The output of this resonant circuit is passed through a discriminator, the variation in frequency is indicated on the screen of CRO. The first method employs an amplitude – modulated signal, while the second method uses a frequency – modulated signal. A simplified diagram of a capacitance transducer circuit is shown in figure.

Capacitance gauges are small in size and they have excellent high – frequency response and high temperature resistance, as well as good linearity resolution and ability to measure both static and dynamic quantities. These gauges are sensitive to temperature, vibrations, have high impedance output and complexity of associated electronic equipment. Dielectric, mounting and clamping difficulties make this gauge not too desirable.

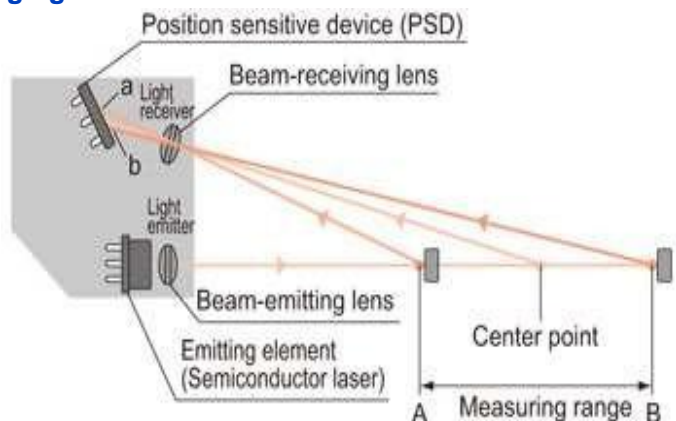
## Laser displacement sensors

### The principle of laser displacement sensor ranging

The principle of laser displacement sensor ranging is a method where triangulation is applied by combining the emitting element and the position sensitive device (PSD) to perform ranging (detecting the amount of displacement).

The emitting element of Devices in laser displacement sensors uses a semi-conductor laser. The laser light is focused through the emitting lens and projected on an object. At that time, some of the light beam that is reflected from the object produces a light spot on the position sensing device. When the object moves, the PSD moves as well. Detecting the changes in positions makes it possible to detect the amount of displacement of the object.

Some of the receiving elements use a linear image sensor, and not the PSD. The PSD enables you to acquire information only about the center position of the amount of light of the entire light spot. On the other hand, the emitting elements with the linear image sensor detect the amount of light received by each cell. Therefore, even when there are variations in the amount of light within the spot due to influences from the object's surface, even more accurate detection can be



performed for the peak position of the light intensity. This significantly reduces errors due to the influence of the objects' surfaces.

## PSD device and position detection methods

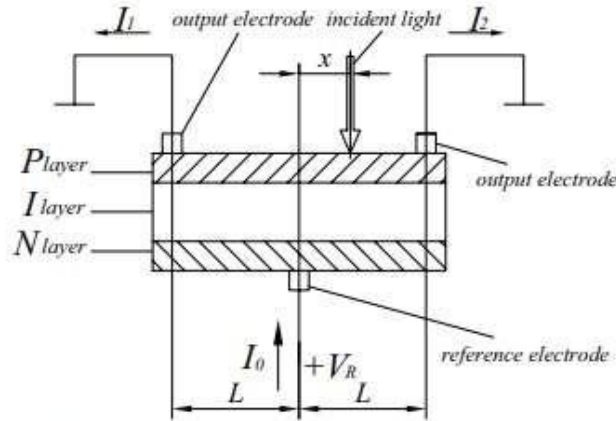
There are two types of PSD device, i.e. linear and area types, or one- and two-dimensional types. The section drawing of linear type PSD is shown in Figure 1. PSD is P-I-N three-layer-structure of Si semiconductor, P-layer is light-sensitive surface, there are two electrodes on its two ends, and the common electrode is connected to reverse voltage. When a light beam is irradiated on light-sensitive surface, lateral electromotive force, will be generated parallel to PN junction. Due to photo effect,

current  $I_0$  is generated by incident light. Because of the existence of lateral electromotive force, current is divided into  $I_1$  and  $I_2$  flowing to the two ends on P-layer,  $I_0 = I_1 + I_2$ . Current distribution is decided by the effective resistors between the position of light spot and the ends. Because resistance on the P-layer is uniformly distributed, so there is the following relationship:

$$\frac{I_1}{I_2} = \frac{L-x}{L+x} \quad (1)$$

$$x = L \cdot \frac{I_2 - I_1}{I_2 + I_1} \quad (2)$$

According to equation (2), the output position signal is only decided by light spot position and has no relationship with light intensity.



**Figure 1.** Sectional drawing of linear type PSD.

There two position detection methods, i.e. the amplitude method and the phase method [1]. The amplitude method determines the position of the incident light beam from difference in the amplitude of the dc currents generated by a steady-state light excitation. The phase detection electronics first convert the currents from the PSD to a voltage, which is then further processed to measure the time difference between the zero crossing points of the sinusoidal outputs, and finally converts the time difference to a voltage which is proportional to the phase difference.

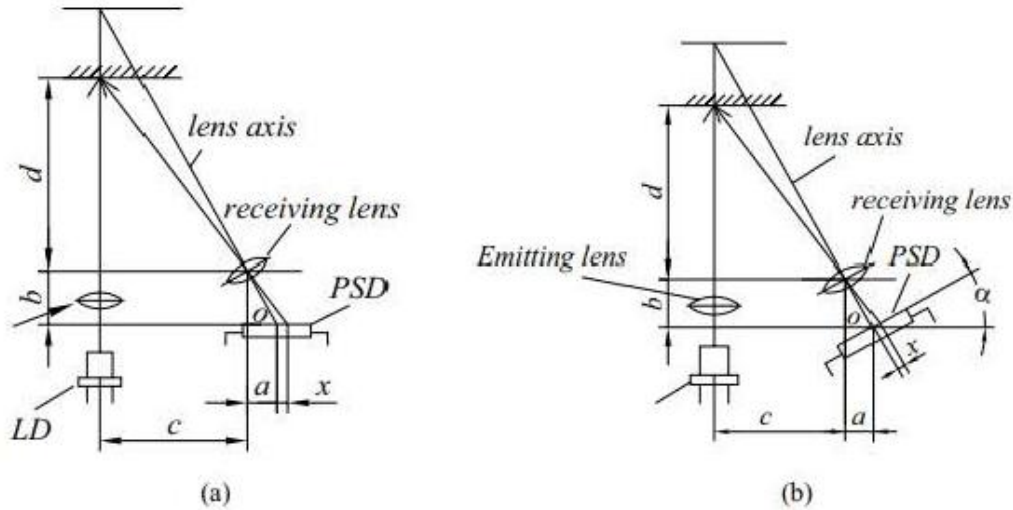
### Dimension determination in structural design and analysis

The configuration or component collocation of PSD based laser displacement sensor has two types, as shown in Figure 2(a) and (b), all of them are based on optical triangulation method. The position of light spot  $x$  can be obtained by detection circuit, and according to the light spot position the distance of the object to be measured can be calculated:

$$d = \frac{b \cdot c}{a + x} \quad (3)$$

$$d = \frac{b \cdot (a + c) \cdot \cos \alpha + b^2 \cdot \sin \alpha}{b \cdot \sin \alpha + x} \quad (4)$$

(3) is for the configuration type of Figure 2(a), and (4) for type of Figure 2(b).



**Figure 2.** Configurations of PSD based laser displacement sensor.

The resolution of the displacement sensor is determined by the sensitive elements and the configuration of the sensor. PSD has continuous sensitive surface, its detection resolution is determined by noise of outside detection circuit and the light-generated current, generally in the range of sub micrometers. As to the configuration of Figure 2(a) and (b), from (3) and (4), there are:

$$\Delta d = -\frac{d^2}{b \cdot c} \cdot \Delta x \quad (5)$$

$$\Delta d = -\frac{d^2}{b \cdot (a+c) \cdot \cos \alpha - b^2 \cdot \sin \alpha} \cdot \Delta x \quad (6)$$

In design, the resolution of the sensor to be developed can be determined with equation (5) and (6). The following equations can be used in dimension determination for configuration of Figure 2(a):

$$L_1 = \sqrt{a^2 + b^2} \cdot \sin^2 \alpha \quad (7)$$

$$L_2 = \sqrt{a^2 + b^2} \cdot \cos^2 \alpha \quad (8)$$

$$f = \sqrt{a^2 + b^2} \cdot \sin^2 \alpha \cdot \cos^2 \alpha \quad (9)$$

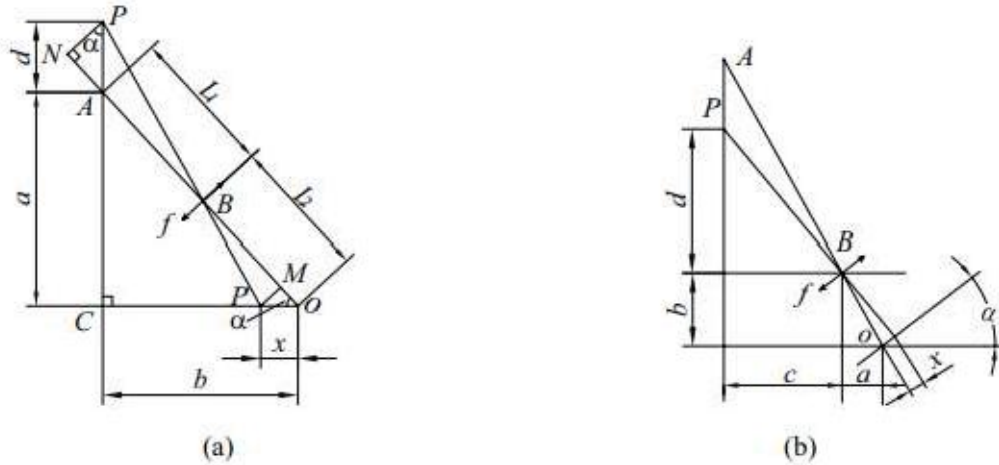


Figure 3. Dimension determination of PSD based laser displacement sensor.

As to the configuration of Figure 2(b), referring to Figure 3(b), the principle of dimension determination is to let the image of reference point *A* be in the sensitive surface of PSD according to lens imaging equation. It should be noted that in practice, the thickness of lens must be considered and accordingly to adjust the installation position of components.

The sensors of both configurations shown in Figure 2 are easy to be fabricated, and also convenient for component installation and adjustment. Configuration of Figure 2(a) has better light spot imaging quality, and when  $\alpha = 45^\circ$  and  $L_1 = L_2$ , in measuring range the light spots can all image in PSD sensitive surface; As to configuration of Figure 2(b), only the reference point images in PSD sensitive surface, other light spots in the measuring range have images out of PSD sensitive surface, but the reflected light rays are nearly perpendicular to PSD sensitive surface and thus more irradiation energy is received. According to equation (5)(6), the resolution gets worse as the measuring distance increases, but configuration of Figure 2(b) has small influence.

